International Journal of Humanities
and Social Sciences (IJHSS)
ISSN(P): 2319-393X; ISSN(E): 2319-3948
Vol. 3, Issue 2, Mar 2014, 235-248

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

"FROM PERCEPTION TO DEDUCTION AND MODELING: THE CHALLENGE OF THE EUCLIDEAN GEOMETRY"

NAIM EL ROUADI[1] & NOHA HUSNI[2]

[1]Associate Professor, University of Balamand, Al-Koura, Lebanon

[2]Instructor, University of Balamand, Al-Koura, Lebanon

ABSTRACT

Reproducing geometric figures that we observe daily is a complex task that students at the age of 14-15 years old face and most of the time are stuck at this point. Panaoura et.al. (2009), confess that such a mission of passing from visualization level of Geometry to the deduction level is difficult. This paper is suggesting a strategy for students to follow in drawing and reproducing Euclidean geometric figures; the strategy is analyzed on Van Hiele Scale and Kuzniak scale to support its validity. This strategy trains the student to use transverse competencies and integrate between multilevel information thus focusing on the 4 concepts of Perception, Deduction, Modeling and Problem Solving. Convinced with the existing challenge, the paper considers an example to justify the profitability of the strategy in a structured sequence.

KEYWORDS: Deduction, Development of Geometry Skills, Euclidean Geometry, Modeling, Problem Solving

INTRODUCTION

Geometric shapes, whether regular or irregular, are everywhere around us in nature (circles in rotational tracks, hexagons in bees hives, the egg shape). Designers of games, movies, buildings, inspired by those shapes reproduce them in their designs (Brown, P. et al, 2011).

Our ancients founded criteria for such geometrical shapes and tried to implement the corresponding forms in many inventions like the projectile launcher, representations of the shadows, constructing the pyramids, astronomy observations…. Nowadays, geometric forms are present in the construction fields (tiles, bricks, windows, doors…), technology and industrial fields (shapes of computers and their accessories, gift boxes) and other fields. Here the 3D is not our concern more than the 2D representations of such designs. For example, top, bottom, and side views are to be drawn in planes for any product to be manufactured or any house to be constructed. Therefore, there is always a need for reproducing geometric figures. It is of utmost importance and necessity that students acquire such a skill before they reach higher educational levels aiming at different professions. Are there specific strategies that can help Grade 9 students (age 14 to 15 years old) to learn how to reproduce accurately geometric forms using simple geometric tools as a compass, a ruler and a set-square? The presented strategy is based on the cognitive approach which considers a set of modeling steps to be followed in order to accomplish a resolution for a complex problem-situation. The objective of this paper is to lead students in Grade 9 - school level to accomplish successfully a geometric task which is reproducing a geometric figure.

In the following approach, we will refer to the theories of Van Hiele, and Kuziniak who discussed how students can develop their geometric skills according to different levels, knowing that Van Hiele approach is linear while Kuzniak approach is not.

The methodological approach is based on the cognitive theory and the related transversal competences.

The G9 students are chosen since they have a long experience in the reasoning of geometry from different resources in the classical Euclidean geometry.

THE PROBLEM SITUATION

This problem (Retrieved Jan 2014) is presented on the website of URL[1].

"Liz places seven pennies in the arrangement shown below: She notices that each of the six outer pennies touches the one in the center and its two outer neighbors"

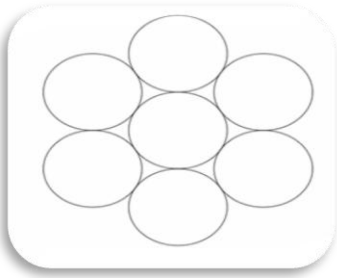

Figure 1

As one can easily notice that any child playing with the 7 identicalpennies or coins can arrange them in the form Liz did. The questions that arise here: can grade 9 students reproduce this figure accurately? What are the geometric basic shapes behind reproducing this figure? What are the geometric skills needed to reproduce this figure using a compass and a ruler or a set square? And what are the successive steps required to form a geometrical strategy? This geometric strategy can be applied to similar cases and slightly amended so that it conforms to other geometric situations.

Figure 2

[1]http://www.illustrativemathematics.org/illustrations/707

THE THEORIES

"I may not be cleverer than the others but I have a method" Rene Descartes (1596 - 1650).

What we are aiming at in this paper falls under the umbrella of the great work achieved by Descartes in his famous book "Geometry" – part I that deals with solving problems regarding the geometry of circles and lines; and the Euclidean geometry which followed the rules and findings of Descartes.

Many other mathematicians and researchers continued looking for solutions in different fields of pure and applied mathematics. For example, Georges Polya indicated in his book "How To Solve It" explicit steps to problem solving; Polya suggested thinking of a plan to recognize something familiar or look for patterns or use similarities. In Cerme 6 – 2009 Panaoura et.al. discussed under 'The Geometric Reasoning Of Primary and Secondary School Students' pointed out the difficulties the students face to move from Geometry I to Geometry II on Kuzniak scale: "*These findings stress the need for helping students progressively move from the geometry of observation to the geometry of deduction."* (Abstract, p1)

In our case, the focus is on 14-15 years old students' abilities to solve Geometry problems. It is not a matter of data or theory acknowledgement that students' may lack more than a matter of a strategy to train them make a smooth shift from what they perceive in a problem to the outcome deductions they can formulate to finally store a model in their long-term memory so that it can be retrieved whenever they are faced with similar problems.

The search for solutions did not stop and will not stop; Pierre Van Hiele and Alain Kuzniak drive our attention towards students' geometric development levels. In what follows the models lined by Van Hiele and Kuzniak are summarized to be adopted as a frame to justify the hypothesis of this paper.

Van Hiele

In 1986, Van Hiele discussed the different levels of students' geometric development. Van Hiele explained that geometric development can be achieved in five sequential levels:

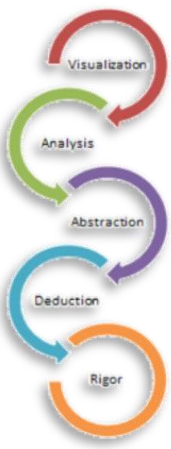

Figure 3

"Visualization" comes first, at this level students can recognize the geometric shapes and name them but only in their standard orientation. At this level students know nothing about the properties of the shapes.

"Analysis" comes next, where students can tell the shape and name of the geometric forms from their properties. At this level the properties are not in order meaning that student cannot recognize that a square is also a rectangle or a parallelogram.

The third level is called "Abstraction", at this level the students can give clear definitions for geometric shapes and they can tell that a rectangle is a parallelogram and not every parallelogram is a rectangle.

The fourth level is "Deduction", at this level the student is able to draw proofs and recognize that there is no unique way to give the proof.

The last level is "Rigor", finally the student is able to relate between geometric concepts in an abstract way thus student can continue the studies from Euclidean to non-Euclidean geometry. (Marchis, 2012)

Experience in class has shown that a student with level one or two on the van Hiele model "often fails in the construction of a geometric configuration which is essential for the solution of the underlying geometric problem - Schumann and Green, 1994",(cited in Patsiomitou, 2009, p2). This true statement is verified every time our students find it difficult to draw geometric shapes in certain problems or they try to reproduce given figure in a specific situation.

Kuzniak

Kuzniak divided the geometry into three levels and insured that there is no linear relation among them but a student can move back and forth between the three levels according to different situations considered. The three levels as perceived in (Kuzniak, A., et al, 2007) can be summarized as such:

"Geometry I" also called "Natural Geometry": Intuition and reality are implemented at this phase of Geometry in order to construct geometric figures using measures and geometric tools.

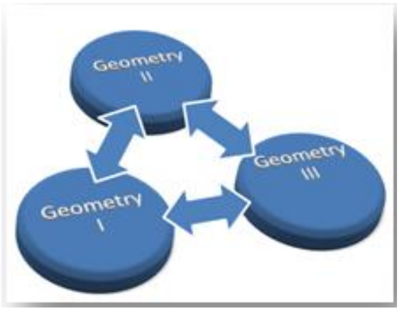

Figure 4

"Geometry II" Also Called Axiomatic Geometry: In this phase, geometry is approached through axioms that are based on real sensory cases or examples. Hypotheses and deductions are also implemented in this phase.

"Geometry III" Also Called the Formal Geometry: This phase is not related to school levels of geometry; it is for university studies in and on geometry. The student can work on geometry away from the reality but formally.

Kuzniak, A., et al, (2007)

As the definitions imply, the strategy suggested is applicable for G9 students, i.e. school level is only under Geometry I and II.

THE APPROACH

The given figure arranged by Liz using the 7 pennies appears to the student as 7 circles; the middle circle is tangent to the six other circles and every circle of the 6-outer circles is tangent to the central circle and the two neighboring circles.

How can we reproduce them? Assume we do not know the radius of the circle although they are identical circles. (of course, if we have the coins we can measure the diameter of one coin and deduce the corresponding radius by dividing by 2, radius = diameter/2). In this problem we are assuming that we do not know the type of the coin and this figure can be reproduced to smaller or larger scales using the same strategy. The main idea lies behind locating the centers of the 7 circles and determining the radius of these circles.

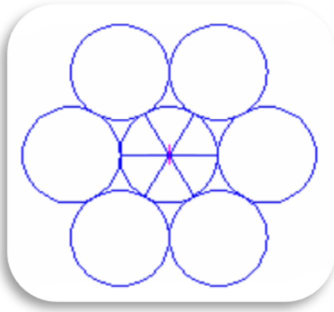

Figure 5

It is important to notice that joining the diametrically opposite points of tangency determines the center of the central circle.

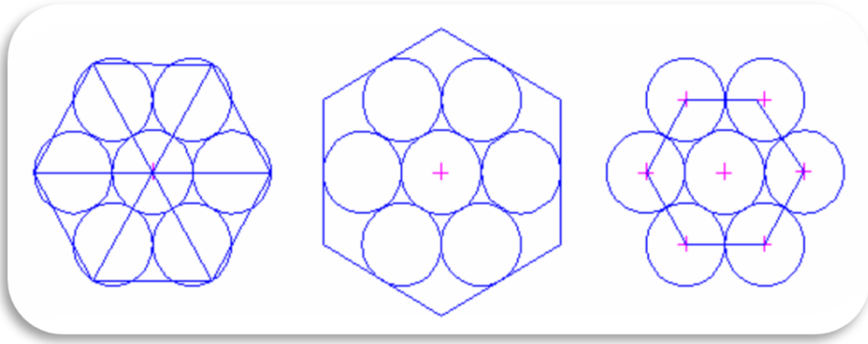

Figure 6

And what's next? We need to locate the centers of the other six circles. So no matter what procedure we follow we'll reach basic construction lines for the figure formed of one or more regular hexagons. Every regular hexagon consists of 6 identical equilateral triangles. Hence the basic geometric shapes required to reproduce Liz's figure are the equilateral triangle and the regular hexagon in addition to the circle itself.

Students in Grade 9 already have the basic skills of the equilateral triangle, the regular hexagon, and the circle as well as the techniques to use the geometric tools: ruler, set square, and the compass to draw them.

This leads us to the axiom: to draw this shape we need to start with a hexagon. Convinced with this axiom, the strategy follows.

OUR STRATEGY

The Approach to reproduce the figure, as mentioned earlier is based on the Descartes strategy: “Dismantle the problem into smaller easy steps”

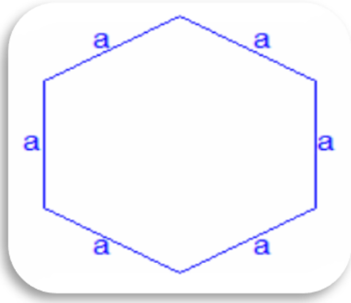

Figure 7

Manipulate with the Given Figure Until You Discover its Basic Geometric Regular Forms

As shown in the previous paragraph Liz’s drawing has the regular hexagon and the equilateral triangle as basic geometric forms.

A hexagon of side “a” is drawn; You may draw the 6-adjacent equilateral triangles but we all know through practice that drawing the regular hexagon is a short cut to the 6-adjacent identical equilateral triangles.

Van Hiele’s Levels:
In this step “Visualization” of Van Hiele’s levels is used. The student can recognize the forms of hexagon and triangle in addition to circle.

Kuzniak Geometric Levels:
According to Kuzniak, this step belongs to “Geometry I” level in which the student uses his intuition for recognizing geometric shapes of circle, triangle and hexagon..

Continue Drawing the Construction Lines You Discovered that Completes the Basics of your Figure

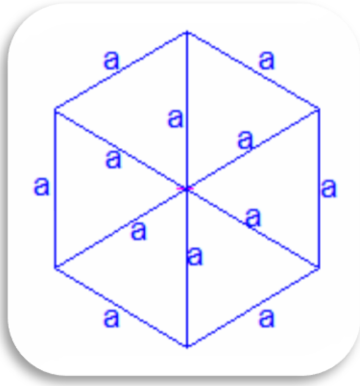

Figure 8

Connect the diametrically opposite vertices; the intersection of which is the center of the central circle. This step divides the hexagon into 6 identical equilateral triangles of side “a”.

Van Hiele's Levels:
In this step "Visualization and Analysis" of Van Hiele's levels are invested. The student not only recognizes the shapes but is able to recognize some of the properties of these shapes. Hence, student is sure that the figure embeds a regular hexagon and equilateral triangles and not any other particular geometric shapes.

Kuzniak Geometric Levels:
According to Kuzniak, this step belongs to "Geometry I" level too in which the student uses his intuition for recognizing geometric shapes and their criteria for example, the hexagon is six identical adjacent equilateral triangles of equal 60° angles and equal sides.

Ask yourself: "How does your Figure Fit in the Construction Lines Drawn?"

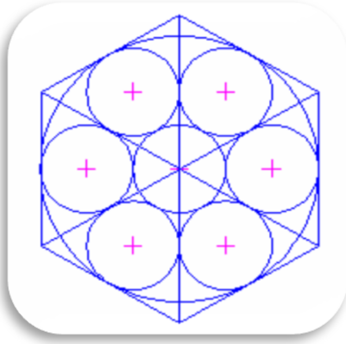

Figure 9

Every circle of the 6-outer circles in inscribed in one of the 6-identical equilateral triangles.

Van Hiele's Levels:
The first three levels are implemented in this step: "Visualization, Analysis, and Abstraction". Student can recognize the 7 identical circles; this is related to their previously acquired knowledge about properties and criteria of circles. Also the property of a circle inscribed in a triangle is needed in this step. In other words, the relations between two geometric forms the triangle and the circle.

Kuzniak Geometric Levels:
According to Kuzniak, this step belongs to "Geometry I and Geometry II" levels as the student is hypothesizing the presence of 6-identical circles and relating them to 6-identical equilateral triangles. Student can deduce that without the triangle, the center of the outer circle cannot be located.

Geometric Criteria and Properties Required to Draw the Regular Geometric Shape you are Supposed to Reproduce

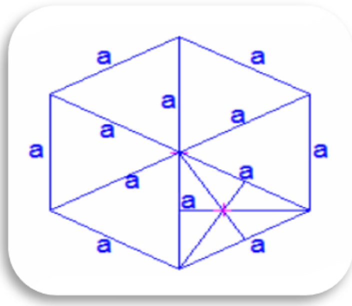

Figure 10

Using the geometry of the figure, the centers of the6–outer circles can be located. The intersection of medians s (or bisectors or height or perpendicular bisectors) is a unique point which is the center of the inscribed circle. Locate the 6 centers of the 6-outer circles.

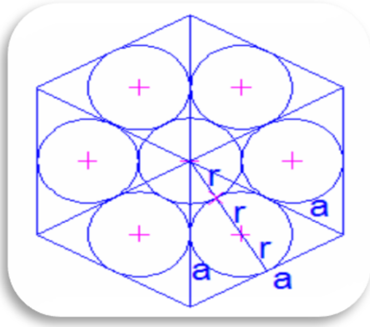

Figure 11

Again from Geometry of the figure, the radius "r"of the7- identical circles can be calculated in terms of "a" as such: in any equilateral triangle of side "a", the height is h = $a\frac{\sqrt{3}}{2}$. On the other hand, geometry of the figure shows: h = 3 x r. Then3r = $a\frac{\sqrt{3}}{2}$ and r = $a\frac{\sqrt{3}}{6}$.

If a = 6 cm, then r = $\sqrt{3}$ cm.This value of $\sqrt{3}$ cm is an irrational number that cannot be measured accurately by a ruler; what is the most accurate way to draw $\sqrt{3}$ cm?

Using right isosceles triangle of side 1cm has a hypotenuse of$\sqrt{2}$cm as per Pythagoras theorem. Now a 1cm side perpendicular to the $\sqrt{2}$cm side can determine another right triangle of hypotenuse $\sqrt{3}$ cm. Use the compass to get the length of the radius and construct the central circle and the 6-outer circles.

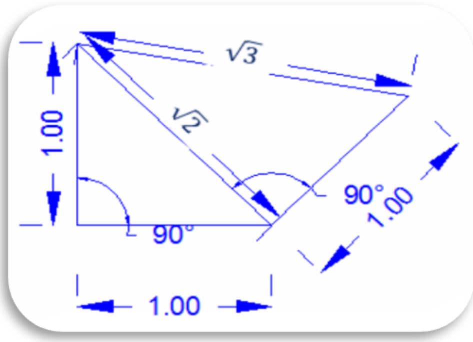

Figure 12

Van Hiele's Levels:
In addition to the first three levels "Visualization, Analysis, and Abstraction", the forth level is implemented in this step "Deduction". The student can draw proofs of the hypotheses he thought of in the previous step. The proof of regular hexagon, the equilateral triangle, the location of the center of the circle inscribed in an equilateral triangle, and the exact measure of an irrational number as $\sqrt{3}$cm

Kuzniak Geometric Levels:
According to Kuzniak, this step belongs to "Geometry II' level since the student is using the deduction part; i.e. carrying out proofs to show that the circle inscribed the triangle is the outer circle required to be drawn in this exercise.

Drawing the Exact Length of an Irrational Number and Consequently the Figure

It is the time to use the geometric tools: the compass to take the exact measure of $\sqrt{3}$ cm to locate the centers of the 6-outer circles and to draw the 7 circles.

Erase the Construction Lines you Used to Draw your Figure

Now the construction lines can be erased to obtain Liz's figure.

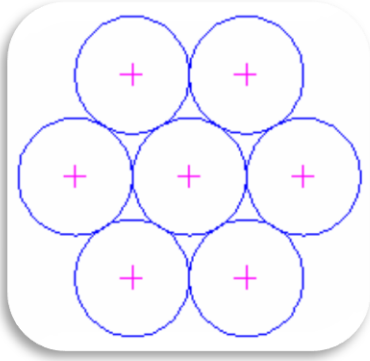

Figure 13

What if you want to Reproduce the Given Figure with a Specific Dimension; Do the Necessary Calculations and Restart Drawing from the Beginning

If you wish to draw the circles of specific radius, for example r = 1.5 cm.

Then the hexagon side a= $6\frac{r}{\sqrt{3}}$= 6 x$\frac{1.5}{\sqrt{3}}$x$\frac{\sqrt{3}}{\sqrt{3}}$ =3 $\sqrt{3}$ cm.

Use the compass to draw 3 times the measure of the irrational number $\sqrt{3}$ cm implementing the same strategy as in step5.5 to construct your hexagon of side a = 3 $\sqrt{3}$ cm. Use the ruler to draw the hexagon and the diametrically opposite diagonals which divides the hexagon into 6 identical equilateral triangles. At this level step 5.3 is done. Proceed from step 5.4 as previously done.

Scientific Observations

An Experiment was done in G9 class of 24 students. The students were divided into groups of 4.The pre-test was to reproduce Liz' figure of the 7 pennies. The results of the pre-test were (as indicated in the table below) as follows: Some students reproduced the figure but not accurately neither to scale and few obtained the accurate figure but not to scale using extended diameters of the central circle and rotations of 60◦ but they did not have a reasoning why they used this way consequently no strategy in mind.

After words, the students were directed to use internet research and they were given the properties presented in the steps of the suggested strategy of this paper (concerning the regular hexagon, the equilateral triangle, the exact length of an irrational number, and the center of circle inscribed in a triangle…), the post-test was given and the groups were asked to reproduce Liz' figure.

The table below shows the results of the post-test as follows: very few still do not have a strategy, and the major part used some strategy or adopted the full strategy to reproduce the figure accurately and to scale.

Table 1

Strategy Test	Absence of Strategy	Using Some Strategy	Strategy Adopted
Pretest percentage	65 %	35 %	0 %
Posttest percentage	12 %	48 %	40 %

The table indicates that the perception influences deduction reasoning to come up with a strategy.

Synthesis

What did our strategy do? Actually, it created a logical relationship among the four concepts: Perception, Deduction (steps followed), Modeling and finally achieving Problem Solving.

Acknowledging that these four concepts formulate the basics of the cognitive approach. This strategy embeds critical thinking (by using comparison between similar problems) and creativity (by integrating the multi-level information in different situations).

This is similar to the work achieved by Duval: "Approaching geometry from a cognitive point of view, he has distinguished four cognitive apprehensions connected to the way a person looks at the drawing of a geometrical figure: perceptual, sequential, discursive and operative (Duval, 1995)." In addition to, "Solving geometrical problems often requires the interactions of these different apprehensions, and "what is called a 'geometrical figure' always associates both discursive and visual representations, (Duval, 2006)" (cited in Panaoura, G., 2009).

The student searches in different resources the ideas and the processes to be adopted in order to finalize his analysis and to cumulate and integrate the new information with the existing one in his mind to formulate a new combination. The new knowledge serves him to formulate a new synthesis. The following image is presented as an illustration of this idea.

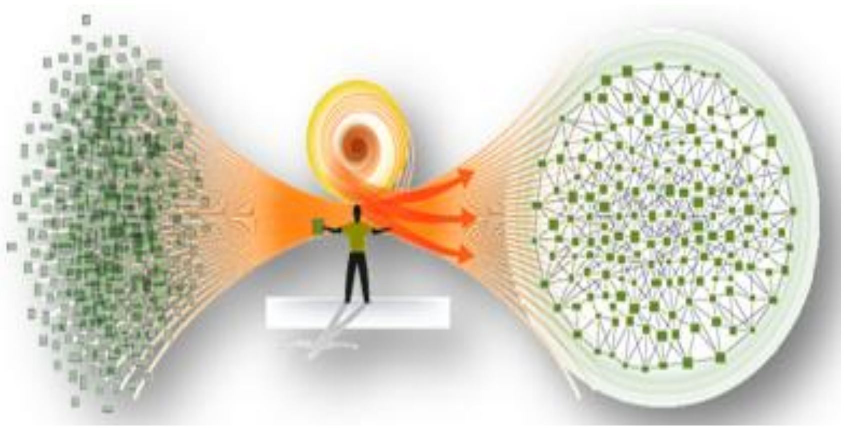

Figure 14

http://www.idiagram.com/ideas/knowledge_integration.html

The student gets his information and in spires different process from working group, its community, the Internet or colleagues in order to integrate with their own knowledge and experience. The image below is a representation of our point.

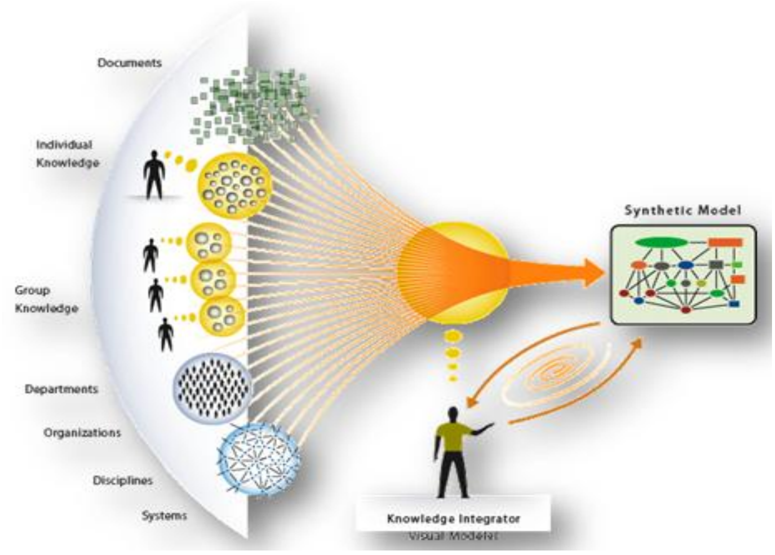

Figure 15

http://www.idiagram.com/ideas/knowledge_integration.html

This strategy has implemented the elements from the cognitive theory: resources, observation, information, documentation, alignment, analysis, deduction and modeling. The two pictures represent show such a strategy can do a huge difference in information processing in our students minds from stacking pieces to connected ones (schema) that are easily retrieved from long term memory whenever needed.

C. Moser (Moser, 2011) presents a taxonomy to illustrate the levels of cross skills necessary for a no vice to go to the professional phase. This taxonomy contains several specific objectives: describe, analyze, try applying rules, argue, act, anticipate, criticize, assume master and engagement.

C. Moser brings these objectives to characterize the change of status from novice to professional. The foregoing presents an important aid for the learner to determine his position with respect to his capacity and his progress in learning. It is used to check the formula

"Action to learn and learn to act."

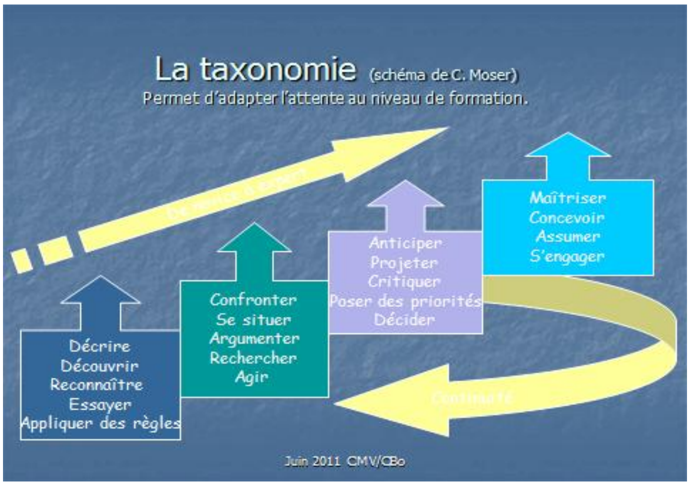

Figure 16

www.hecvsante.ch/.../évaluer_dans_un_système_par_competences

CONCLUSIONS

The "Decision strategies international: DSI" represents a long training as the shape shown in the taxonomy of cross skills (C. Moser).The following image illustrates the necessary elements for the learner to master a rational strategy. This allows the student to meet the challenges resulting from the resolution of a problem.

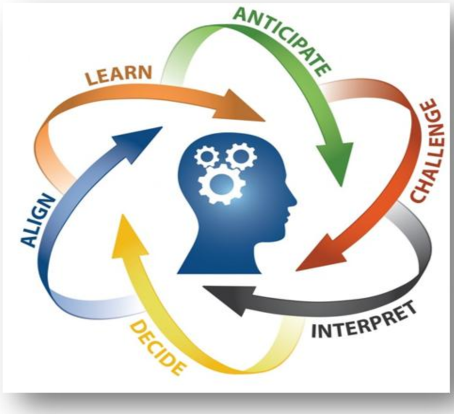

Figure 17

"Our *Strategic Aptitude Assessment*™ unpacks *The 6 Key Elements of Strategic Thinking*™ to help leaders identify and improve their strategic approach to succeed in more than one future.

The 6 Key Elements of Strategic Thinking include

- **Anticipate:** proactively monitoring the market environment and changes inside and outside your industry
- **Challenge:** questioning organizational and industry-wide assumptions that others take for granted
- **Interpret:** connecting multiple data points in new and insightful ways to make sense of complex, ambiguous situations
- **Decide:** seeking multiple options to ensure flexible decision-making
- **Align:** engaging stakeholders to understand change readiness, manage differences and create buy-in
- **Learn:** continuously reflecting on successes and failures to improve performance and decision-making"

http://www.decisionstrat.com/insights/strategic-aptitude-assessment/

Finally, we can say that the perception contains in itself the appearance of an analysis of information in correspondence with some knowledge after a convincing interpretation is done to make a decision followed by devolution. Thus, mental representation is built to form some modeling of new knowledge.

REFERENCES

1. Brown, P., Evans, M., Hunt, D., McIntosh, J., Pender, B., Ramagge, J. (2011): 'Introduction To Plane Geometry - A Guide For Teachers – Years 7-8, Measurement and Geometry Module 9', *The Improving Mathematics Education in Schools (TIMES) Project – June 2011.*

2. Decision Strategies International (DSI) (2014), 'Strategic Aptitude Assessment' http://www.decisionstrat.com/insights/strategic-aptitude-assessment/
3. HESAV (Haute École De Santé Vaud), 2011 : 'Evaluer dans un système par compétences', *presentation of reference 23.06.11, Juin 2011 CMV/C Bo.*
4. Kuzniak,A., Gagatsis, A., Ludwig, M., Marchini, C.: (2007), 'From Geometric Thinking Knowledge to Geometric Work', *Proceedings of Cerme 5, WG7 Report.*
5. Marchis, I.: (2012), 'Preservice Primary School Teachers' Elementary Geometry Knowledge', *Acta Didactica Napocensia,* ISSN 2065-1430, vol 5, number 2, 2012.
6. Panaoura, G., Gagatsis, A. (2009): 'The Geometrical Reasoning of Primary And Secondary School Students' *Proceedings of CERME 6.*
7. Patsiomitou, S., Emvalotis, A.: (2009), 'Developing Geometric Thinking Skills Through Dynamic Diagram Transformations' - *6th Mediterranean Conference on Mathematics Education*
8. Polya, G., (renewed 1973) 'How to solve it' A new Aspect of Mathematical Approach – Stanford University- 2nd edition *Princeton University Press – Princeton, New Jersey*
9. "The Geometry of Rene Descartes", translated by Smith, D.E., and Latham, M., *Dover Publications Inc. New York, 10 N.Y*